

THE BAYESIAN STOCHASTIC VOLATILITY MODEL EXAMPLE

This example is borrowed from the web-site of Prof. Medibert Freitas Lopes (Institute of Education and Research, São Paulo, Brazil).

The raw time series data are:

p_t \equiv closing price of the Deutscher Aktienindex (German stock index, with acronym "DAX") on day t .

for 1860 consecutive (trading) days during 1991-1999.

Let $r_t \equiv \log\left(\frac{p_t}{p_{t-1}}\right) =$ return on day t .

Then let

$$y_t \equiv r_t - \bar{r}, \quad \bar{r} \equiv \text{average of the } r_t\text{'s}$$

The Bayesian stochastic volatility model in the rstan-based script of Prof. Freitas Lopes is:

$$y_t = \exp\left(\frac{1}{2} h_t\right) \varepsilon_t$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma u_t$$

$$h, \mu, \sigma, \phi \sim N\left(\mu, \frac{\sigma^2}{1-\phi}\right),$$

$$\varepsilon_t \stackrel{\text{ind}}{\sim} t_\nu(0, 1) \quad \left(\begin{array}{l} \text{Student } t\text{-dist}^\square \\ \text{with } \nu \text{ degrees} \\ \text{of freedom} \end{array} \right)$$

$$u_t \sim N(0, 1)$$

Priors: $\mu \sim N(0, 100)$, $\sigma^2 \sim \text{Gamma}\left(\frac{1}{2}, \frac{1}{2}\right)$

$$\nu \sim \text{Gamma}(1, 0.05),$$

$$\phi \sim N(0.8, 0.09) \text{ truncated to } [-1, 1].$$

