37457

Advanced Bayesian Methods

Missing Data and Measurement Error Models

Logistic Penalized Spline model

If the data were complete:

$$y_i \overset{\text{ind.}}{\sim} \text{Bernoulli}\left(\frac{1}{1+\exp\{-f(x_i)\}}\right)$$

$$y_i = \begin{cases} 1 & \text{if diabetes for } i\text{th Pima Indian} \\ 0 & \text{otherwise} \end{cases}$$

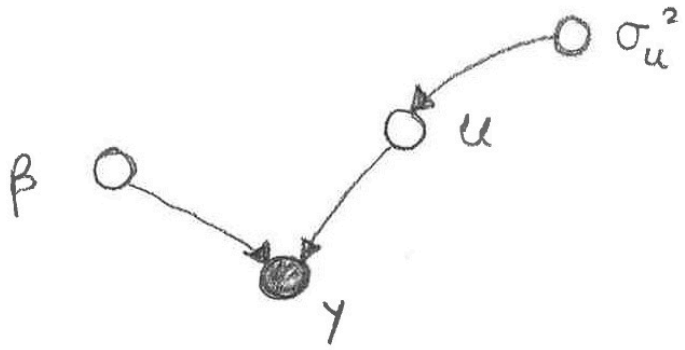
$$x_i = \text{body mass index of } i\text{th Pima Indian}$$

$$f(x) = \beta_0 + \beta_1 + \sum_{k=1}^{K} u_k z_k(x), \quad u_k \overset{\text{ind.}}{\sim} N(0, \sigma_u^2).$$

Pima Indians Diabetes Data

body mass index	indicator of diabetes
33.6	1
26.6	0
23.3	1
31.0	1
35.3	0
30.5	1
NA	1
37.6	0
38.0	1
27.1	0
.	.
.	.
.	.
42.0	1
29.7	0
28.0	0
39.1	1
NA	0
19.4	0
24.2	0
.	.
.	.
.	.
30.1	1
30.4	0

Directed Acyclic Graph for Ordinary (Complete Data) Model



The Simplest Missing Data Model

Define:

$$r_i = \begin{cases} 1 & \text{if body mass index observed for } i\text{th Pima Indian} \\ 0 & \text{if body mass index missing for } i\text{th Pima Indian} \end{cases}$$

$$r_i|\rho \overset{\text{ind.}}{\sim} \text{Bernoulli}(\rho).$$

This is known as a

missing completely at random model

Caveat of Previous Model

Missingness often depends on the predictor (or response) variables.

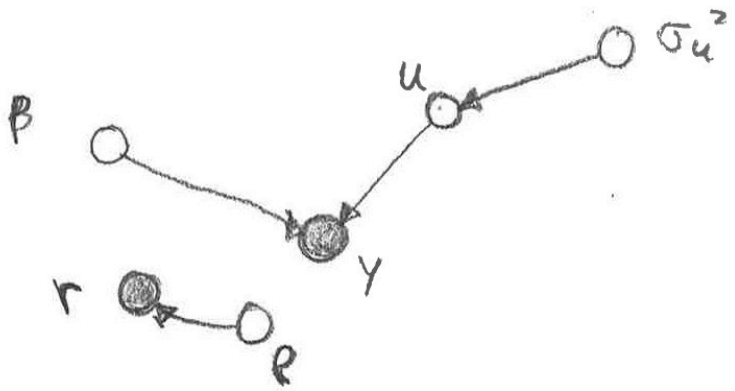
e.g. if asked "Are you a cigarette smoker?" in a survey, whether or not you answer may depend on smoking status.

More sophisticated missing data model for the Pima Indians data:

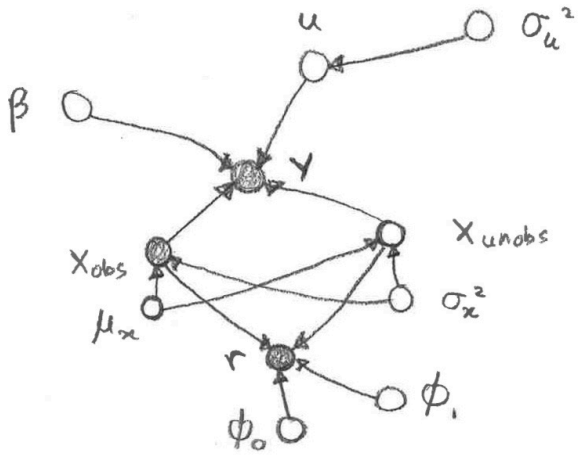
$$r_i \overset{\text{ind.}}{\sim} \text{Bernoulli}\left(\frac{1}{1+\exp\{-(\phi_0+\phi_1 x_i)\}}\right)$$

where the missingness is not at random.

Directed Acyclic Graph for Missing Completely at Random Model



Directed Acyclic Graph for Missing Not at Random Model



Look at and then run PIDana.R

Coronary Heart Disease Example

$$y_i = \begin{cases} 1 & \text{if coronary heart disease for } i\text{th patient} \\ 0 & \text{otherwise} \end{cases}$$

x_i = low density lipoprotein cholesterol level of ith patient

v_i = age in years of ith patient

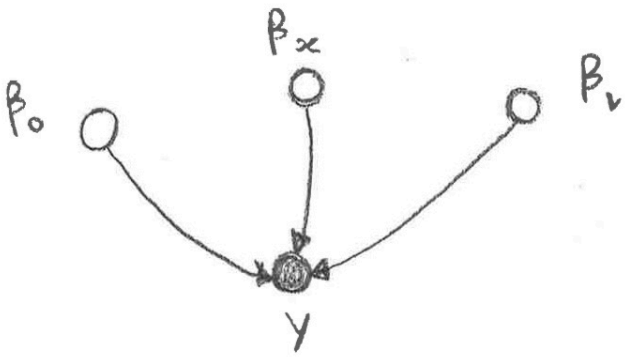
Ideal (Bayesian logistic regression) model:

$$y_i|\beta_0, \beta_x, \beta_v \overset{\text{ind.}}{\sim} \text{Bernoulli}\left(\frac{1}{1+\exp\{-(\beta_0 + \beta_x\, x_i + \beta_v\, v_i)\}}\right)$$

One Final Topic

MEASUREMENT ERROR MODELS

Corresponding Directed Acyclic Graph



Cost of Data Collection

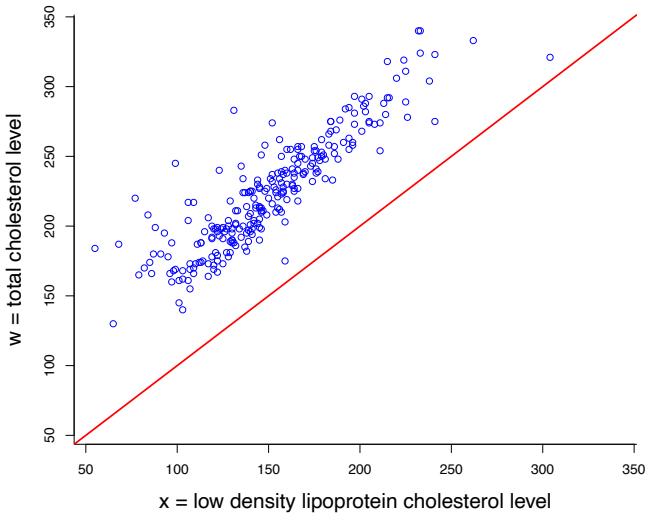
Suppose that

$$x \equiv \text{low density lipoprotein cholesterol level}$$

costs \$2,500 to measure for each patient, but

$$w \equiv \text{total cholesterol level}$$

costs \$50 to measure for each patient.



Options (with Limited Budget)

- Only use x with a smaller sample of, say, about 30 people.

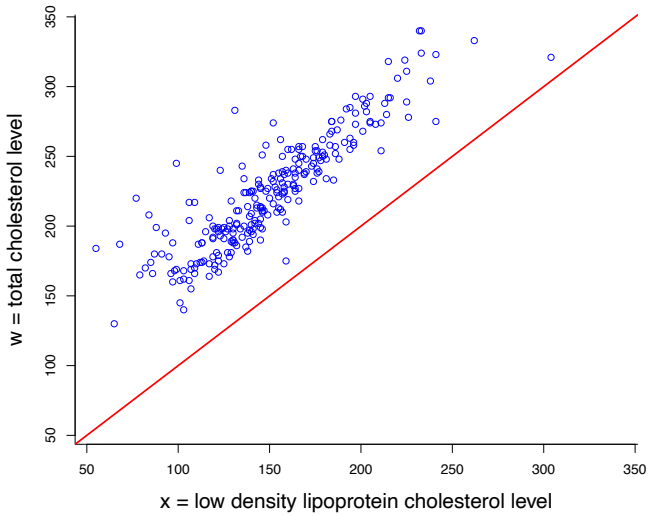
  $\Longrightarrow$ lower power to detect effect of x.

- Only use w with a large sample of, say, 1500 people.

  $\Longrightarrow$ hoping that w is a good surrogate for x, which may not be reasonable.

- Model the relationship between w and x using a relatively small validation data set and incorporate this into the model with all w data (large) and x data (small).

  $\Longrightarrow$ measurement error model!



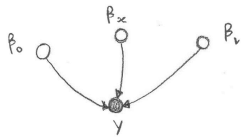
Measurement Model Add-On

$$w_i|x_i, \phi_0, \phi_1, \sigma_w^2 \sim N(\phi_0 + \phi_1 x_i, \sigma_w^2),$$

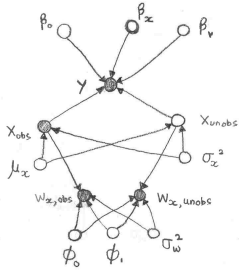
$$x_i|\mu_x, \sigma_x^2 \overset{\text{ind.}}{\sim} N(\mu_x, \sigma_x^2),$$

with the x_is only partially observed.

Both Directed Acyclic Graphs Together

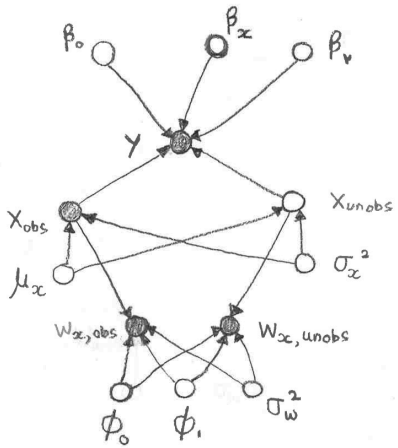


Ordinary Model



Measurement Error Model

Directed Acyclic Graph for Measurement Error Model



Look at CHDmeaErrAna.R

Results for Effect on Coronary Heart Disease

Using the fancy measurement error model we get the effect of low density lipoprotein cholesterol level as follows:

$\widehat{\beta}_x = 1.11$ with 95% credible interval $(0.0289, 2.31)$

i.e. a statistically significant effect

If we had just used total cholesterol data then we would have gotten

$\widehat{\beta}_w = 0.556$ with 95% credible interval $(-0.065, 1.22)$

lack of significance; not using the better predictor.

THE END OF

37457 SPRING 2022

MATERIAL

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