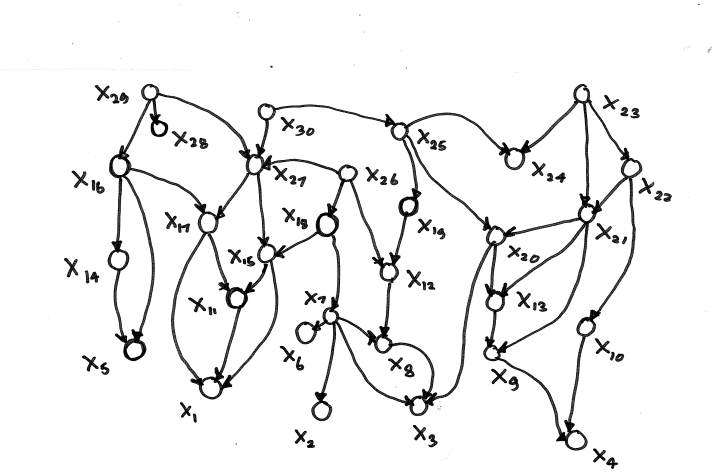
37458

Advanced Bayesian Methods

Professor Matt Wand

University of Technology Sydney



Distribution Theory and Uni Stats Subjects

1st subject	single random variable X	density function $f_X(x)$
2nd subject	two random variables X, Y	density function $f_{X,Y}(x, y)$
this subject	many random variables maybe even hundreds of r.v.s	density function $f_{X_1,\ldots,X_k}(x_1, \ldots, x_k)$ $k \approx 300$, say

With dozens, maybe even hundreds, of random variables useful to have a way to keep their distributional structure organised.

$\Longrightarrow$ PROBABILISTIC GRAPHS

Streamlined Density Function Notation

First subjects in Stats:

X is a random variable.

$f_X(x)$ is probability mass function (X discrete) or density function (X continuous) at x.

$f_{X,Y}(x, y)$ is joint density function of X, Y.

$X = x$ corresponds to X having observed value x.

Remainder of this subject (in keeping with advanced stats literature):

x is a random variable.

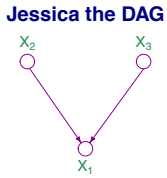
$p(x)$ is density function of x at x (admitted abuse!).

$p(x, y)$ is joint density function of x, y.

$x = \mathring{x}$ corresponds to x having observed value $\mathring{x}$.

Hand out alternative Assignment 1.

Do You Remember...



Not to be confused with...

Important to Always Keep in Mind

We are just doing Distribution Theory

just like you did in earlier subjects

and Assignment 1!!

Jessica the PROBABILISTIC DAG

$$p(x_1, x_2, x_3) = \prod_{i=1}^{3} p(x_i|\text{parents of } x_i) = p(x_1|x_2, x_3)\, p(x_2)\, p(x_3)$$

SPECIFIC EXAMPLE:

$$p(x_1|x_2, x_3) = \frac{2x_1 + 14x_2 + 5x_3}{252 + 18x_1 + 45x_3}, \quad x_1, x_2, x_3 = 1, 2, 3$$

$$p(x_2) = \frac{3 + 7x_2^2}{107}, x_2 = 1, 2, 3$$

$$p(x_3) = \frac{12}{13(1 + x_3)}, x_3 = 1, 2, 3$$

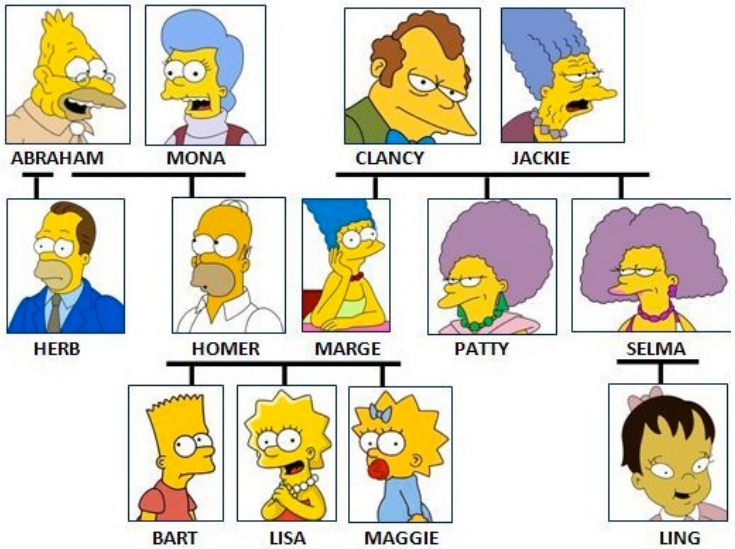
$$\Longrightarrow \; p(x_1, x_2, x_3) = \frac{2x_1 + 14x_2 + 5x_3}{252 + 18x_1 + 45x_3} \times \frac{3 + 7x_2^2}{107} \times \frac{12}{13(1 + x_3)}, \quad x_1, x_2, x_3 = 1, 2, 3$$

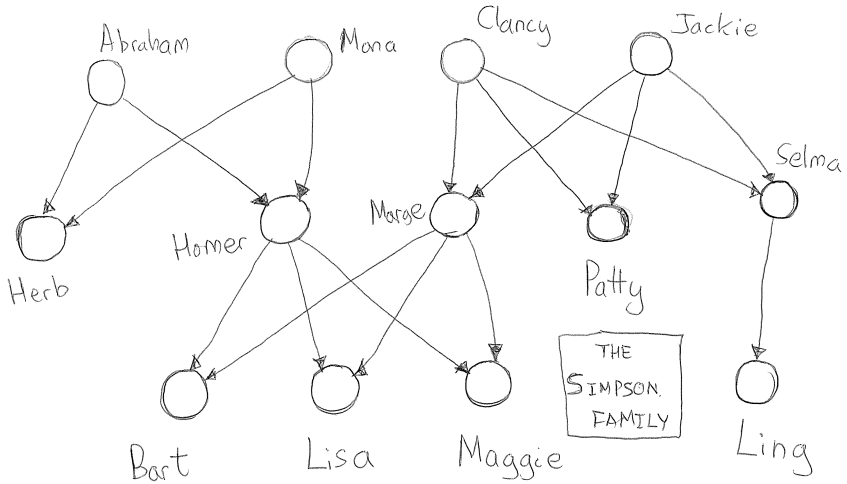
SECOND SPECIFIC EXAMPLE (ANOTHER Jessica the PROBABILISTIC DAG)

$$x_1|x_2,x_3 \sim N(x_2,x_3), \quad x_2 \sim \text{Beta}(7,9), \quad x_3 \sim \text{Gamma}(7,19)$$

$$p(x_1|x_2,x_3) = (2\pi x_3)^{-1/2}\exp\{-(x_1-x_2)/(2x_3)\}$$
$$p(x_2) = 45045\, x_2^6(1-x_2)^8, \quad 0<x_2<1.$$
$$p(x_3) = \frac{7^{19}}{\Gamma(19)}x_3^6\exp(-19x_3), \quad x_3>0.$$

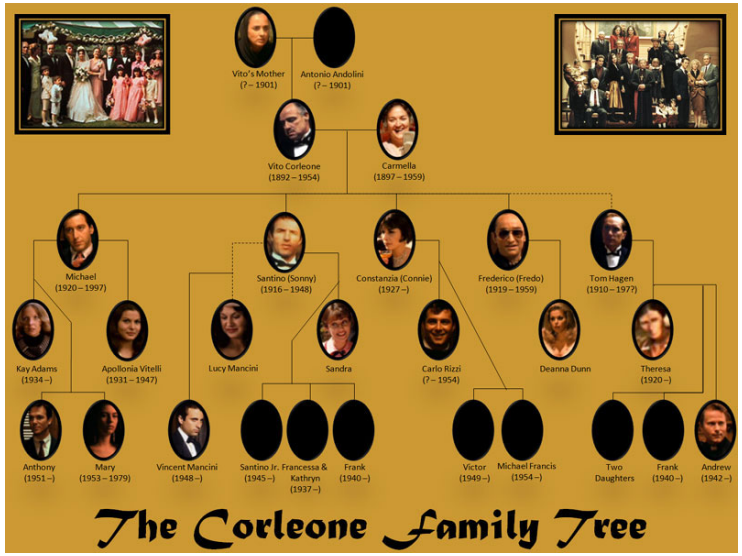
$$\implies p(x_1,x_2,x_3) = (2\pi x_3)^{-1/2}\exp\{-(x_1-x_2)/(2x_3)\}\times 45045\, x_2^6(1-x_2)^8$$
$$\times\frac{7^{19}}{\Gamma(19)}x_3^6\exp(-19x_3), \quad 0<x_2<1,\ x_3>0$$





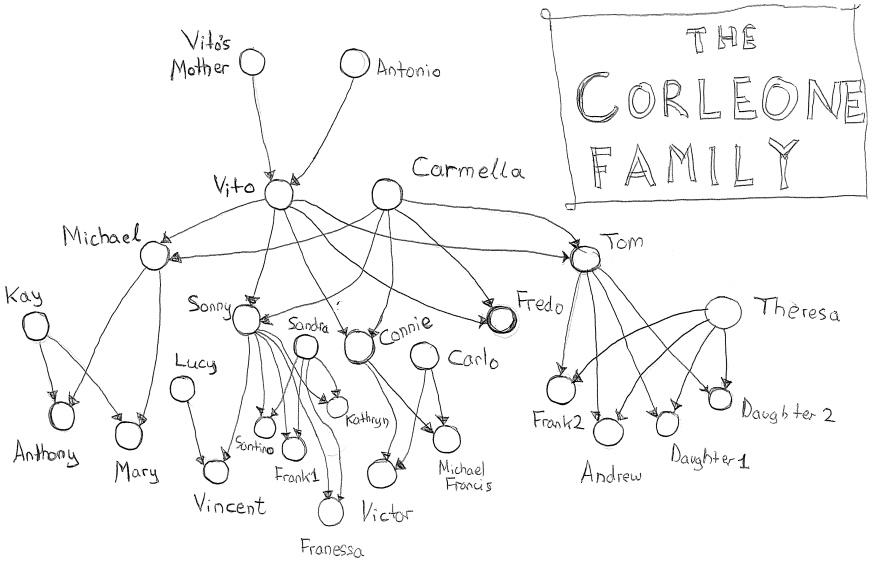
Do smallest ancestral sub-graph exercise with

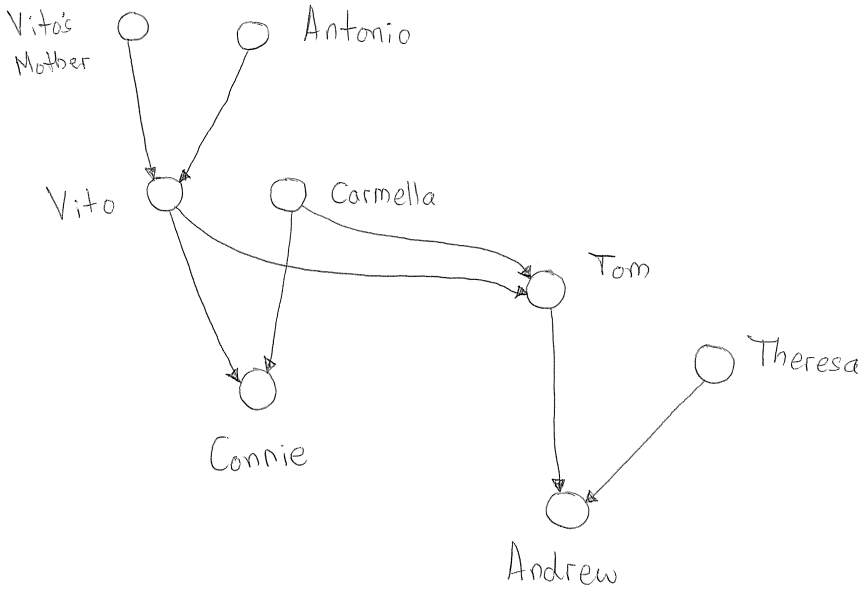
two randomly chosen Simpson family members.



Class Exercise

Draw the smallest ancestral sub-graph containing Connie and Andrew.





Graph Theoretic Concepts

- Maximal cliques.
- Moralisation.
- Small ancestral sub-graphs.
- Separation.
- Markov blankets.

You need to be on top of all of the above from Chapter 1 of the *Graph Theory and Statistics Notes* (which you will now start reading obsessively).

Hand out Assignment 2.

Housekeeping

Getting set up for Laboratory 1 in the second half of Class 3.

See new link on subject web-site.