37458

Advanced Bayesian Methods

Professor Matt Wand

University of Technology Sydney

Distribution Theory and Uni Stats Subjects

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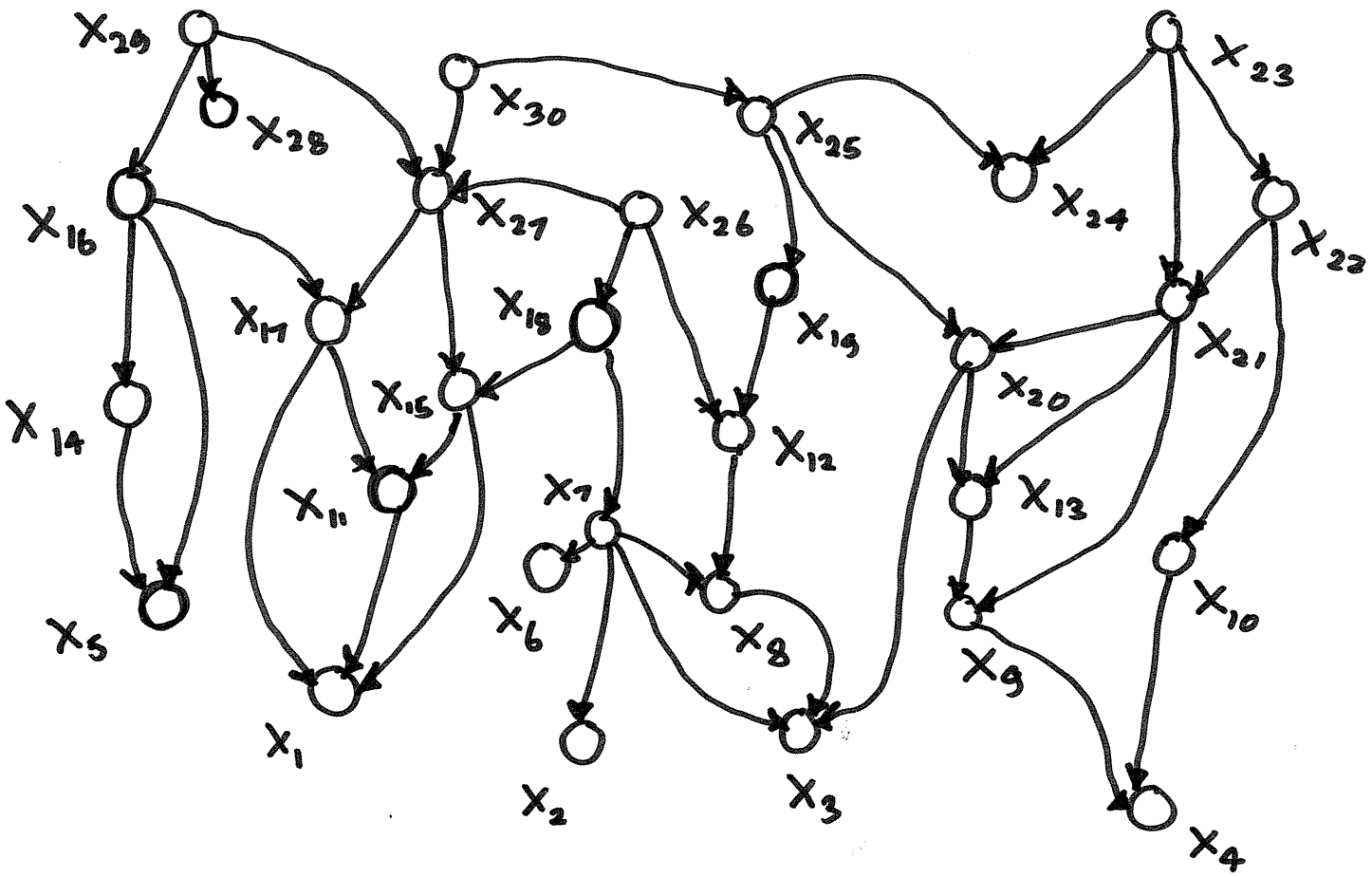
$\Longrightarrow$ PROBABILISTIC GRAPHS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x_{7}
x_{13}
x_{10}
x_{5}
x_{6}
x_{8}
x_{9}
x_{1}
x_{2}
x_{3}
x_{4}

Streamlined Density Function Notation

First subjects in Stats:

X is a random variable.

$f_X(x)$ is probability mass function (X discrete) or density function (X continuous) at x.

$f_{X,Y}(x, y)$ is joint density function of X, Y.

$X = x$ corresponds to X having observed value x.

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Remainder of this subject (in keeping with advanced stats literature):

x is a random variable.

$p(x)$ is density function of x at x (admitted abuse!).

$p(x, y)$ is joint density function of x, y.

$x = \mathring{x}$ corresponds to x having observed value $\mathring{x}$.

Hand out alternative Assignment 1.

Important to Always Keep in Mind

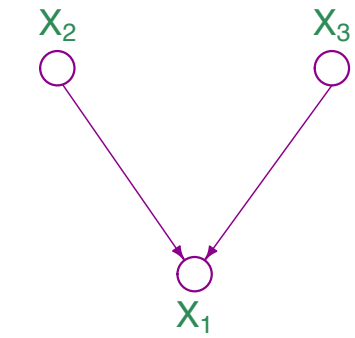
We are just doing Distribution Theory

just like you did in earlier subjects

and Assignment 1!!

Do You Remember...

Jessica the DAG



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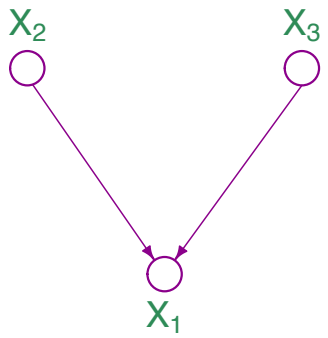


Not to be confused with...

Jessica the PROBABILISTIC DAG

$$p(x_1, x_2, x_3) = \prod_{i=1}^{3} p(x_i|\text{parents of } x_i) = p(x_1|x_2, x_3)\, p(x_2)\, p(x_3)$$

Jessica the PROBABILISTIC DAG

$$p(x_1, x_2, x_3) = \prod_{i=1}^{3} p(x_i|\text{parents of } x_i) = p(x_1|x_2, x_3)\, p(x_2)\, p(x_3)$$

SPECIFIC EXAMPLE:

$$p(x_1|x_2, x_3) = \frac{2x_1 + 14x_2 + 5x_3}{252 + 18x_1 + 45x_3}, \quad x_1, x_2, x_3 = 1, 2, 3$$

$$p(x_2) = \frac{3 + 7x_2^2}{107}, x_2 = 1, 2, 3$$

$$p(x_3) = \frac{12}{13(1 + x_3)}, x_3 = 1, 2, 3$$

$$\Longrightarrow \quad p(x_1, x_2, x_3) = \frac{2x_1 + 14x_2 + 5x_3}{252 + 18x_1 + 45x_3} \times \frac{3 + 7x_2^2}{107} \times \frac{12}{13(1 + x_3)}, \quad x_1, x_2, x_3 = 1, 2, 3$$

SECOND SPECIFIC EXAMPLE (ANOTHER Jessica the PROBABILISTIC DAG)

$$x_1|x_2, x_3 \sim N(x_2, x_3), \quad x_2 \sim \text{Beta}(7, 9), \quad x_3 \sim \text{Gamma}(7, 19)$$

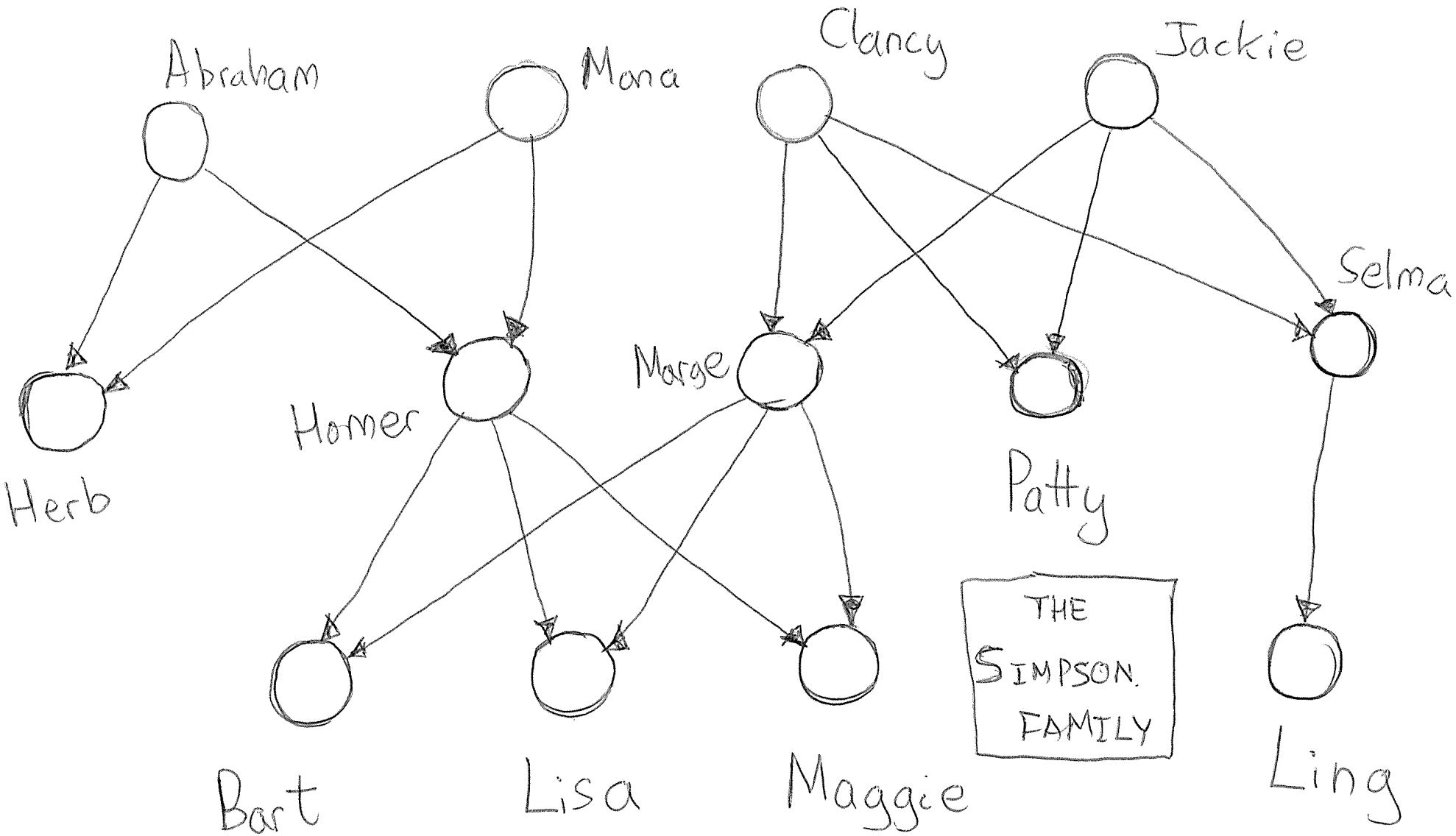
SECOND SPECIFIC EXAMPLE (ANOTHER Jessica the PROBABILISTIC DAG)

$$x_1|x_2, x_3 \sim N(x_2, x_3), \quad x_2 \sim \text{Beta}(7, 9), \quad x_3 \sim \text{Gamma}(7, 19)$$

$$p(x_1|x_2, x_3) = (2\pi x_3)^{-1/2} \exp\{-(x_1 - x_2)/(2x_3)\}$$
$$p(x_2) = 45045\, x_2^6 (1 - x_2)^8, \quad 0 < x_2 < 1.$$
$$p(x_3) = \frac{7^{19}}{\Gamma(19)} x_3^6 \exp(-19x_3), \quad x_3 > 0.$$

$$\implies p(x_1, x_2, x_3) = (2\pi x_3)^{-1/2} \exp\{-(x_1 - x_2)/(2x_3)\} \times 45045\, x_2^6 (1 - x_2)^8$$
$$\times \frac{7^{19}}{\Gamma(19)} x_3^6 \exp(-19x_3), \quad 0 < x_2 < 1,\ x_3 > 0$$


ABRAHAM
MONA
CLANCY
JACKIE
HERB
HOMER
MARGE
PATTY
SELMA
BART
LISA
MAGGIE
LING


Abraham
Mona
Clancy
Jackie
Selma
Herb
Homer
Marge
Patty
THE SIMPSON FAMILY
Bart
Lisa
Maggie
Ling

Do smallest ancestral sub-graph exercise with

two randomly chosen Simpson family members.

Vito's Mother
(? – 1901)

Antonio Andolini
(? – 1901)

Vito Corleone
(1892 – 1954)

Carmella
(1897 – 1959)

Michael
(1920 – 1997)

Santino (Sonny)
(1916 – 1948)

Constanzia (Connie)
(1927 –)

Frederico (Fredo)
(1919 – 1959)

Tom Hagen
(1910 – 197?)

Kay Adams
(1934 –)

Apollonia Vitelli
(1931 – 1947)

Lucy Mancini

Sandra

Carlo Rizzi
(? – 1954)

Deanna Dunn

Theresa
(1920 –)

Anthony
(1951 –)

Mary
(1953 – 1979)

Vincent Mancini
(1948 –)

Santino Jr.
(1945 –)

Francessa &
Kathryn
(1937 –)

Frank
(1940 –)

Victor
(1949 –)

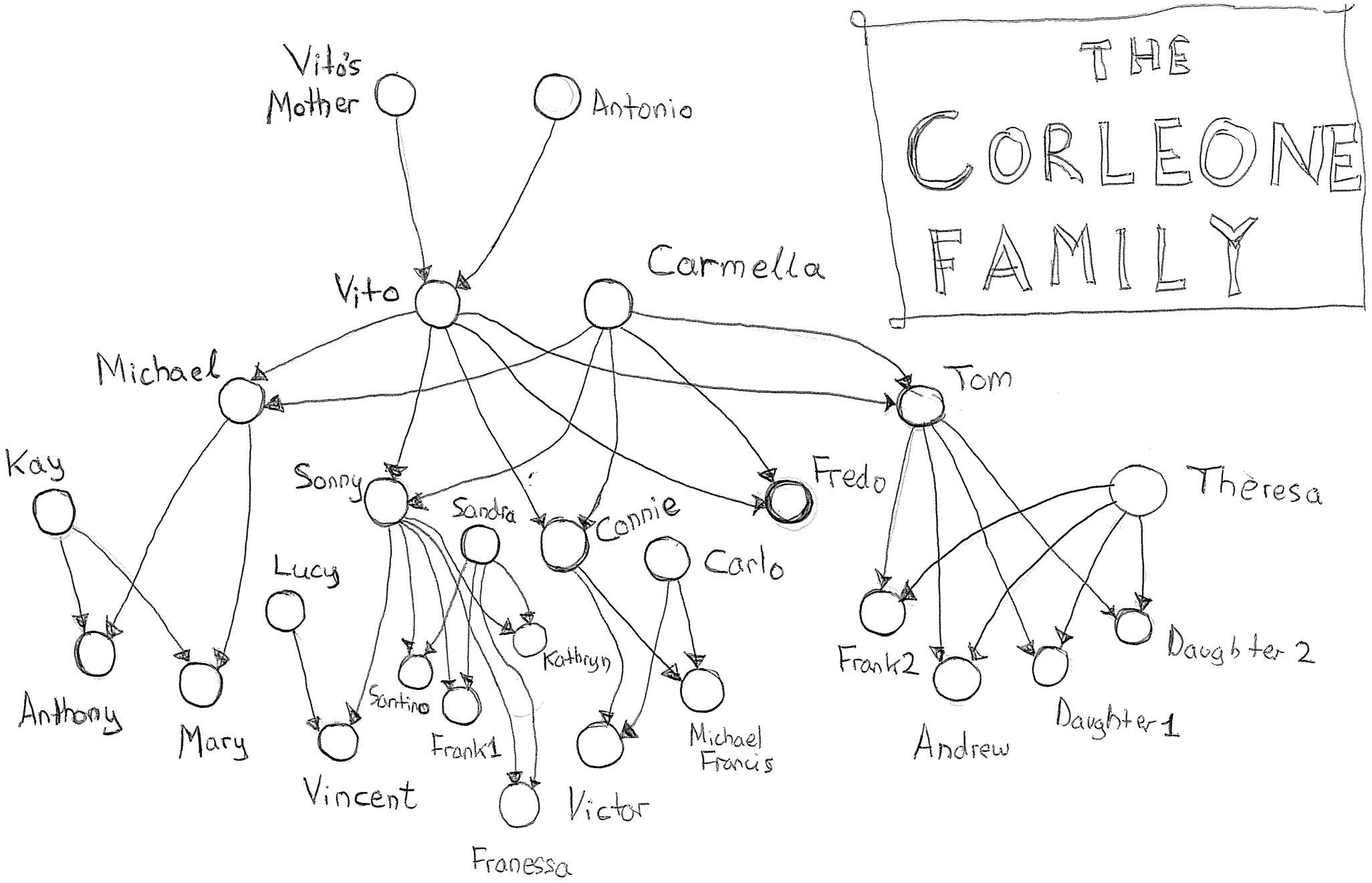
Michael Francis
(1954 –)

Two
Daughters

Frank
(1940 –)

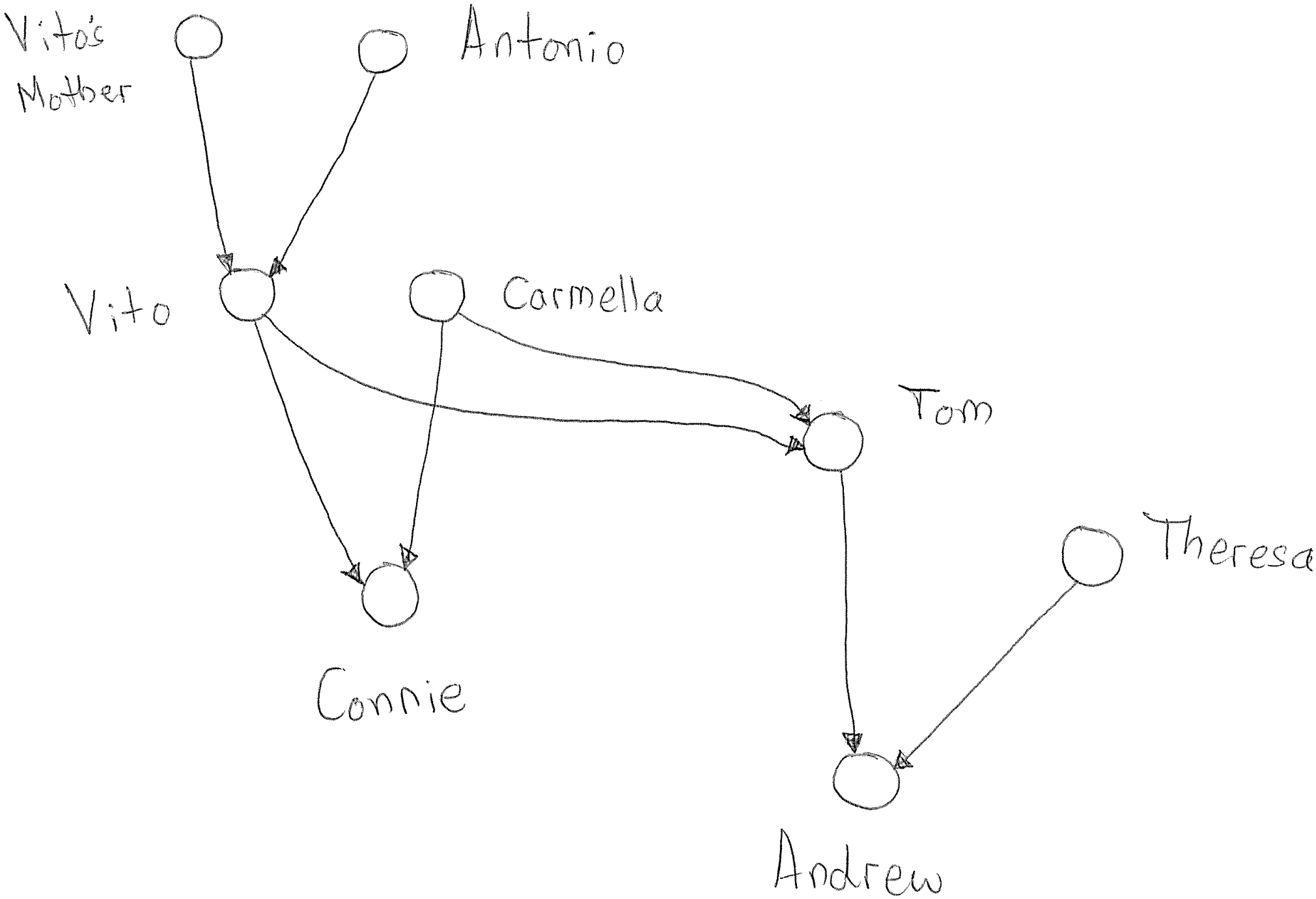
Andrew
(1942 –)

The Corleone Family Tree


THE
CORLEONE
FAMILY
Vito's Mother
Antonio
Vito
Carmella
Michael
Tom
Kay
Sonny
Fredo
Sandra
Connie
Theresa
Lucy
Carlo
Kathryn
Anthony
Santino
Frank2
Daughter 2
Mary
Frank1
Daughter 1
Vincent
Michael Francis
Andrew
Victor
Franessa

Class Exercise

Draw the smallest ancestral sub-graph containing Connie and Andrew.


Vito's Mother
Antonio
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Graph Theoretic Concepts

- Maximal cliques.
- Moralisation.
- Small ancestral sub-graphs.
- Separation.
- Markov blankets.

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You need to be on top of all of the above from Chapter 1 of the *Graph Theory and Statistics Notes*
(which you will now start reading obsessively).

Hand out Assignment 2.

Housekeeping

Getting set up for Laboratory 1 in the second half of Class 3.

See the relevant link on subject web-site.