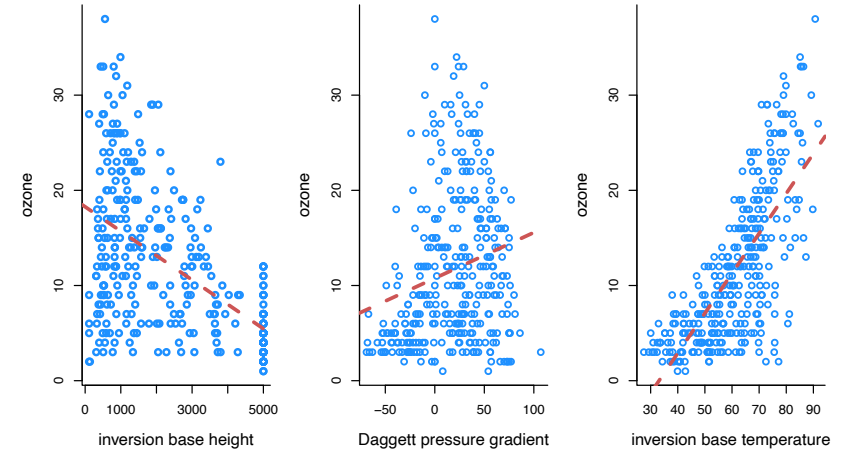


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Advanced Bayesian Methods

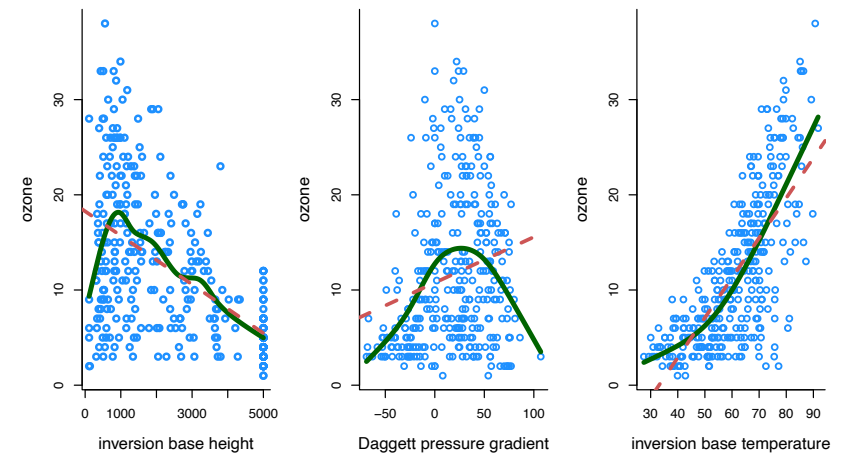
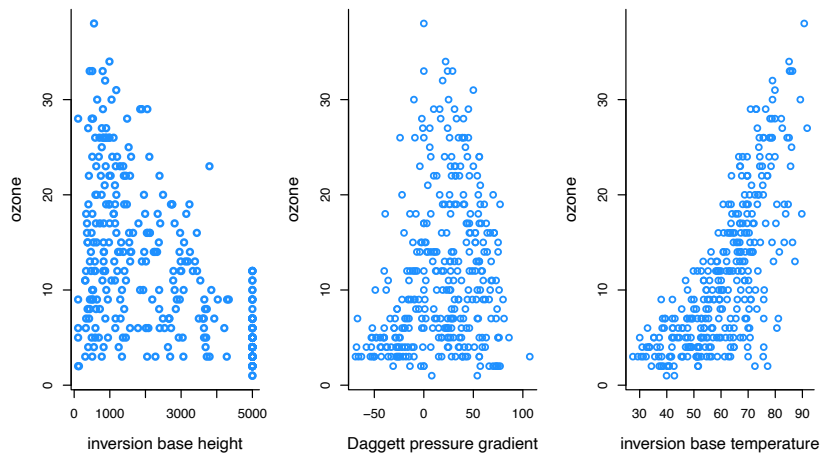
Generalized Additive Models

Linear Model Fits



Data from Ozone Study from California, U.S.A.

Generalized Additive Model Improvement



Linear Model Versus (Generalized) Additive Model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$$

$$y_i = \beta_0 + f_1(x_{1i}) + f_2(x_{2i}) + f_3(x_{3i}) + \varepsilon_i$$

f_1, f_2 and f_3 are unspecified "smooth" functions chosen by the data.

Extension to Three Predictors

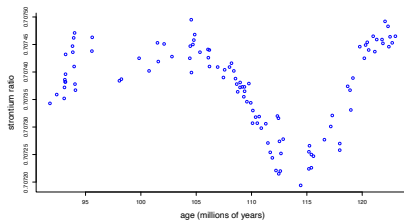
$$f_1(x_1) = \beta_1 x_1 + \sum_{k=1}^{K_1} u_{1k} z_{1k}(x_1), \quad u_{1k} | \sigma_{u1}^2 \stackrel{\text{ind.}}{\sim} N(0, \sigma_{u1}^2),$$

$$f_2(x_2) = \beta_2 x_2 + \sum_{k=1}^{K_2} u_{2k} z_{2k}(x_2), \quad u_{2k} | \sigma_{u2}^2 \stackrel{\text{ind.}}{\sim} N(0, \sigma_{u2}^2),$$

$$f_3(x_3) = \beta_3 x_3 + \sum_{k=1}^{K_3} u_{3k} z_{3k}(x_3), \quad u_{3k} | \sigma_{u3}^2 \stackrel{\text{ind.}}{\sim} N(0, \sigma_{u3}^2),$$

Tricking Mixed Models into Penalized Spline Fitting

First consider the single predictor case:

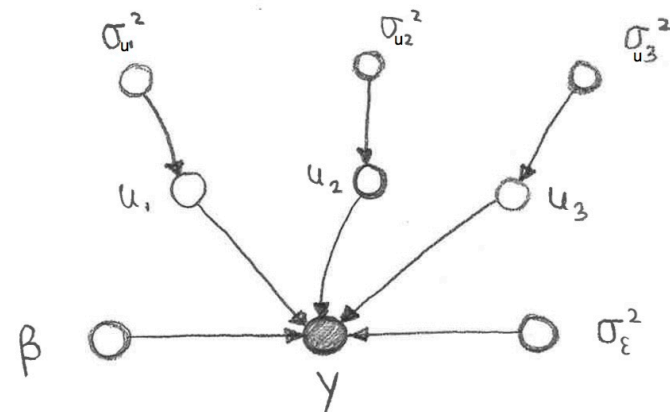


Spline model: $y_i = f(x_i) + \varepsilon_i$ where $f(x) = \beta_0 + \beta_1 x + \sum_{k=1}^K u_k z_k(x)$.

The $z_1(\cdot), \dots, z_K(\cdot)$ are suitable spline basis functions.

TRICK TO AVOID OVERFITTING: $u_k \stackrel{\text{ind.}}{\sim} N(0, \sigma_u^2)$ (Sec. 2.7 of pen. spline notes)

Just Another Bayesian Model with this DAG



Laboratory 3 later today shows how to fit using Stan.

Bayesian versus Non-Bayesian GAM Analysis

Look at ozoneCalifViaStan.R

As we will see in Laboratory 3, the Bayesian inference engine approach is much slower.

HOWEVER the Bayesian engine approach:

- has more accurate inference for binary response data,
- can handle arbitrary response distributions,
- allows non-constant variances,
- can handle missing data properly.

Laboratory 4 will provide illustration in a few weeks from now.

Non-Bayesian Generalized Additive Model Software

Most generalized additive model analyses in R are non-Bayesian, with packages such as:

- **gam**
- **mgcv**
- **VGAM**
- **gamlss**

Laboratory 3 contains illustration using **mgcv**.

Generalized Additive Model Selection

MOTIVATION

The Sydney real estate data (within the R package **HRW** and part of **Laboratory 4**) has data on

price of house
and

and 38 other variables for 37,676 houses in Sydney

IMPORTANT PRACTICAL QUESTION

How to select a good generalized additive model
from the 38 candidate predictors?

37457 Adv. Baye. Meth.

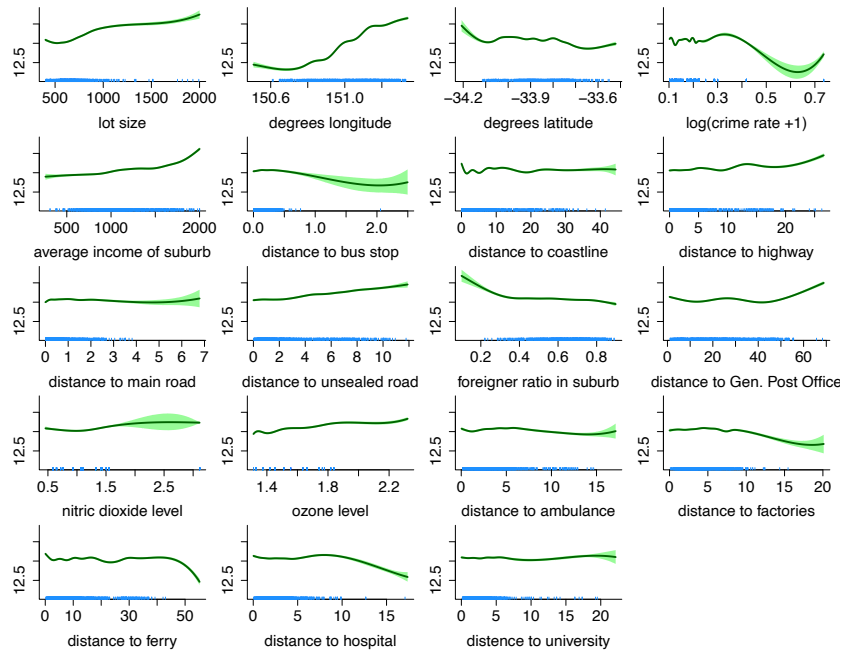
MEETS

current doctoral research

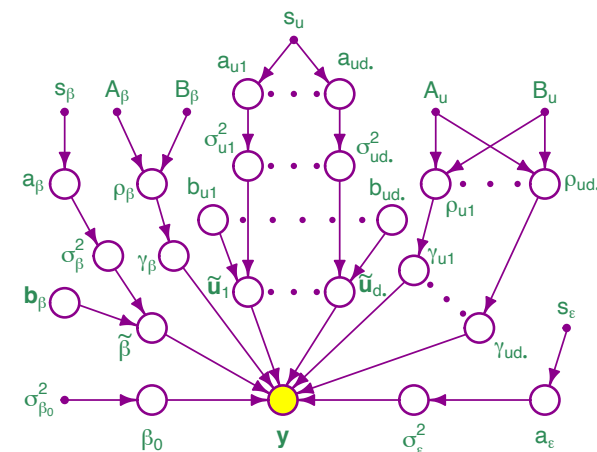
i.e. VIRGINIA'S THESIS

Generalized additive model selection using
Virginia X. He and Matt P. Wand current research via R:

```
install.packages("gamselBayes")  
library(gamselBayes)  
gamselBayesVignette()
```



The gamselBayes DAG



BUT NOTE: rjags or rstan very slow \implies we coded MCMC ourselves in C++

ASIDE: Bayesian Analysis with Multiple Chains

All JAGS and Stan analyses to date have used

single Markov chain Monte Carlo samples (“single chains”)

It is often advocated to use

multiple chains

and then the

Brooks-Gelman-Rubin convergence diagnostic.

See pages 50-51 of the *Penalized Spline* notes...

$$9/3=3$$

⇒ it's time for **Laboratory 3**.

