UNIVERSITY OF TECHNOLOGY, SYDNEY

School of Mathematical Sciences

37457 Advanced Bayesian Methods

ASSIGNMENT 1 WITH STREAMLINED NOTATION

Due time and date: No, you don't have to hand this one in.

1. Let x be a discrete random variable taking values 0 and 1 with equal probability and y be another random variable such that

$$\begin{aligned} p(y|x=0) &= \frac{y}{3}, \quad y = 1, 2, \\ p(y|x=1) &= \frac{5-y}{10}, \quad y = 1, 2, 3, 4. \end{aligned}$$

   (a) Construct a table for the joint probability function $p(x, y)$.

   (b) Find $P(x + y \leq 2)$.

   (c) Find $p(y)$, the marginal probability mass function of y.

2. Let discrete random variables x and y have joint probability mass function

$$p(x,y) = \begin{cases} \dfrac{221(3y+2)}{603(9x^2+4)}, & x = 1, 2, 3, \quad y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

   Determine the marginal probability mass functions $p(x)$ and $p(y)$.

3. Let continuous random variables x and y have joint density function

$$p(x,y) = \begin{cases} \dfrac{y-x}{105}, & 2 < x < 5, \quad 5 < y < 12, \\ 0, & \text{otherwise} \end{cases}$$

   Determine the marginal density functions $p(x)$ and $p(y)$.

Please turn over...

4. Let continuous random variables x and y have joint density function

$$p(x,y) = \begin{cases} \dfrac{e^{-y(x^2+1)}}{\pi}, & y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the marginal density functions $p(x)$ and $p(y)$.

5. Suppose that continuous random variables x and y have joint density function satisfying

$$p(x,y) \propto \exp\left(13xy - 94x^2 - \tfrac{1}{2}y^2\right), \quad -\infty < x < \infty,\ -\infty < y < \infty.$$

(The $\propto$ notation is relatively standard throughout the mathematical sciences and means that the left-hand side equals the right-hand side except for multiplicative factors that do not depend on the function arguments. For example, if $g(x,y) = 171\cos(x+12y)$ then we may write $g(x,y) \propto \cos(x+12y)$.)

Determine $p(y|x = \mathring{x})$, the conditional density function of y given $x = \mathring{x}$.