UNIVERSITY OF TECHNOLOGY SYDNEY

School of Mathematical and Physical Sciences

37458 Advanced Bayesian Methods

ASSIGNMENT 1

Due time and date: 10:05am, 22nd March, 2019 assuming that the class meeting time is decided to be 10:00-12:00 on Fridays.

Submission method: Hand to Professor Wand at start of class.

NOTE: For the benefit of participants requiring assistance with this assignment, a help session will be held at 4pm-5pm on Thursday 21st March 2019 in Room CB07.05.008. If you require assistance but have a class clash with this time then please e-mail Professor Wand via `matt.wand@uts.edu.au`.

1. Let X be a discrete random variable taking values 0 and 1 with equal probability and Y be another random variable such that

$$\begin{aligned} f_{Y|X}(y|0) &= \frac{y}{3}, \quad y = 1, 2, \\ f_{Y|X}(y|1) &= \frac{5-y}{10}, \quad y = 1, 2, 3, 4. \end{aligned}$$

   (a) Construct a table for the joint probability mass function $f_{X,Y}(x, y)$.

   (b) Find $P(X + Y \leq 2)$.

   (c) Find $f_Y(y)$, the marginal probability mass function of Y.

2. Let discrete random variables X and Y have joint probability mass function

$$f_{X,Y}(x, y) = \begin{cases} \dfrac{75(7y+2)}{416(11x^2+1)}, & x = 1, 2, 3, \quad y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

   Determine the marginal probability mass functions $f_X(x)$ and $f_Y(y)$.

3. Let continuous random variables X and Y have joint density function

$$f_{X,Y}(x, y) = \begin{cases} \dfrac{y-x}{105}, & 2 < x < 5, \quad 5 < y < 12, \\ 0, & \text{otherwise} \end{cases}$$

   Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.

Please turn over...

4. Let continuous random variables X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{e^{-y(x^2+1)}}{\pi}, & -\infty < x < \infty, \quad y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.

5. Suppose that continuous random variables X and Y have joint density function satisfying

$$f_{X,Y}(x,y) \propto \exp\left(13xy - 94x^2 - \tfrac{1}{2}y^2\right), \quad -\infty < x < \infty, \ -\infty < y < \infty.$$

(The $\propto$ notation is relatively standard throughout the mathematical sciences and means that the left-hand side equals the right-hand side except for multiplicative factors that do not depend on the function arguments. For example, if $g(x,y) = 171\cos(x+12y)$ then we may write $g(x,y) \propto \cos(x+12y)$.)

Determine $f_{Y|X}(y|x)$, the conditional density function of Y given $X = x$.

♦ ♦ ♦