

UNIVERSITY OF TECHNOLOGY SYDNEY
School of Mathematical and Physical Sciences
37457 Advanced Bayesian Methods

ASSIGNMENT 1

Due time and date: 3:05pm, 12th August, 2022 assuming that the class meeting time is decided to be 3:00-5:00 on Fridays. Otherwise, the due time and date is at the start of Class 2.

Submission method: Hand to Professor Wand at start of class.

NOTE: For the benefit of participants requiring assistance with this assignment, a help session will be held at 2pm-3pm on Thursday 11th August 2022 in Room 006, Level 6, Building 7.

1. Let X be a discrete random variable taking values 0 and 1 with equal probability and Y be another random variable such that

$$\begin{aligned}f_{Y|X}(y|0) &= \frac{y}{3}, \quad y = 1, 2, \\f_{Y|X}(y|1) &= \frac{5-y}{10}, \quad y = 1, 2, 3, 4.\end{aligned}$$

- (a) Construct a table for the joint probability mass function $f_{X,Y}(x, y)$.
 - (b) Find $P(X + Y \leq 2)$.
 - (c) Find $f_Y(y)$, the marginal probability mass function of Y .
2. Let discrete random variables X and Y have joint probability mass function

$$f_{X,Y}(x, y) = \begin{cases} \frac{75(7y+2)}{416(11x^2+1)}, & x = 1, 2, 3, \quad y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine the marginal probability mass functions $f_X(x)$ and $f_Y(y)$.

3. Let continuous random variables X and Y have joint density function

$$f_{X,Y}(x, y) = \begin{cases} \frac{y-x}{105}, & 2 < x < 5, \quad 5 < y < 12, \\ 0, & \text{otherwise} \end{cases}$$

Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.

Please turn over...

4. Let continuous random variables X and Y have joint density function

$$f_{X,Y}(x, y) = \begin{cases} \frac{e^{-y(x^2+1)}}{\pi}, & -\infty < x < \infty, \quad y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.

5. Suppose that continuous random variables X and Y have joint density function satisfying

$$f_{X,Y}(x, y) \propto \exp\left(13xy - 94x^2 - \frac{1}{2}y^2\right), \quad -\infty < x < \infty, \quad -\infty < y < \infty.$$

(The \propto notation is relatively standard throughout the mathematical sciences and means that the left-hand side equals the right-hand side except for multiplicative factors that do not depend on the function arguments. For example, if $g(x, y) = 171 \cos(x + 12y)$ then we may write $g(x, y) \propto \cos(x + 12y)$.)

Determine $f_{Y|X}(y|x)$, the conditional density function of Y given $X = x$.

