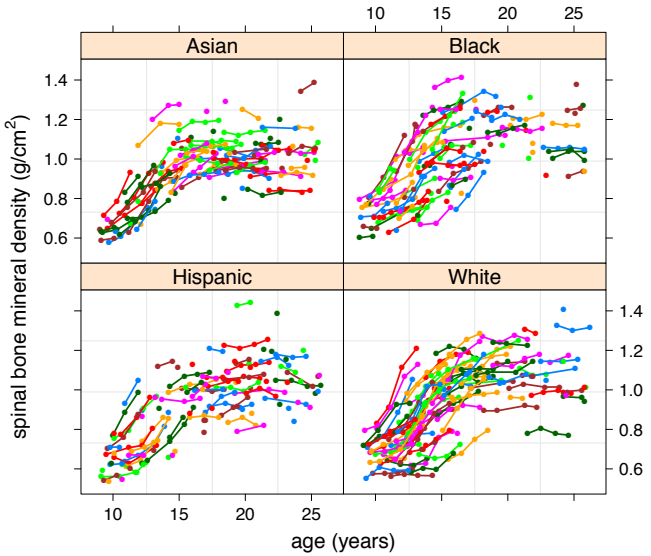
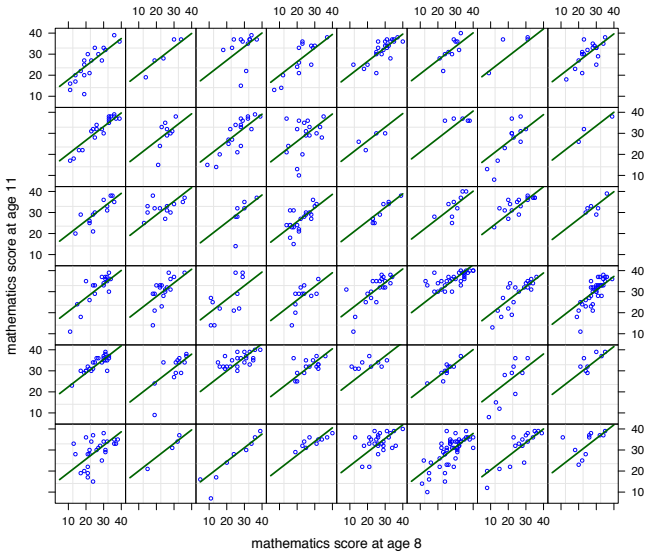
37458

Advanced Bayesian Methods

Some Generalized Additive Models Extensions





Generalized Additive Mixed Model Extension

SBMD = spinal bone mineral density

Ordinary generalized additive model:

$$\text{SBMD}_i = f(\text{age}_i) + \beta_2\,\text{Black}_i + \beta_3\,\text{Hispanic}_i + \beta_4\,\text{White}_i + \varepsilon_i$$

But this ignores the longitudinal (repeated measures) aspect of the data.

$\Downarrow$

Generalized additive mixed model:

$$\text{SBMD}_{ij} = u_{\text{grp},i} + f(\text{age}_{ij}) + \beta_2\,\text{Black}_i + \beta_3\,\text{Hispanic}_i + \beta_4\,\text{White}_i + \varepsilon_{ij},$$
$$u_{\text{grp},i} \overset{\text{ind.}}{\sim} N(0, \sigma^2_{\text{grp}}).$$

Treating Grouped and Penalized Splines Together

$$Z = \left[\begin{array}{c|c} \text{random} & \text{spline} \\ \text{intercept} & \text{basis} \\ \text{indicators} & \text{functions} \end{array}\right]$$

$$\text{covariance matrix of random effects vector} = \begin{bmatrix} \sigma^2_{\text{grp}} \boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2_{\text{spl}} \boldsymbol{I} \end{bmatrix}$$

σ^2_{grp} = controls between-group variability

σ^2_{spl} = controls amount of spline penalization

See Assignment 7 for further details and Stan implementation.

Bivariate Function Extensions

Generalized additive model with three predictors:

$$y_i = \beta_0 + f_1(x_{1i}) + f_2(x_{2i}) + f_3(x_{3i}) + \varepsilon_i$$

A bivariate function extension:

$$y_i = \beta_0 + f_{12}(x_{1i}, x_{2i}) + f_3(x_{3i}) + \varepsilon_i$$

Often (but not necessarily) the (x_{1i}, x_{2i}) data correspond to geographical position (e.g. (longitude,latitude)).

This extension also known as a geoadditive model.

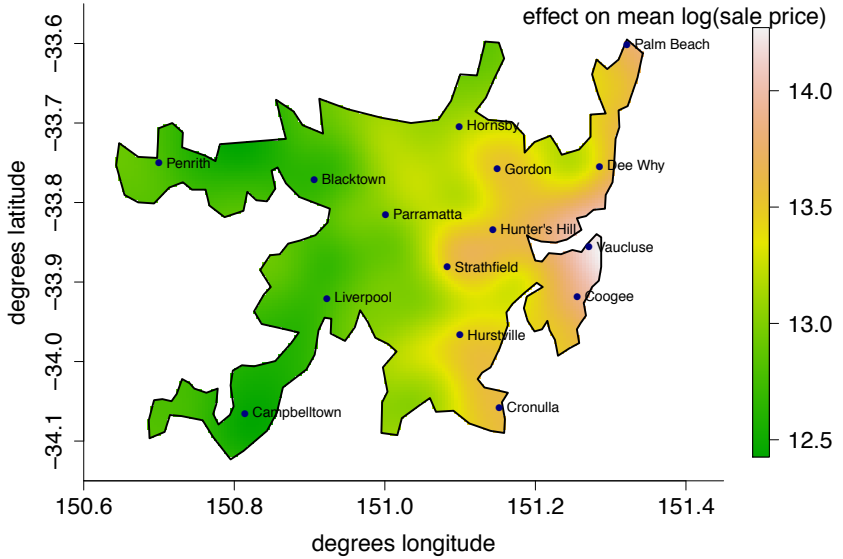
Sydney Real Estate Example

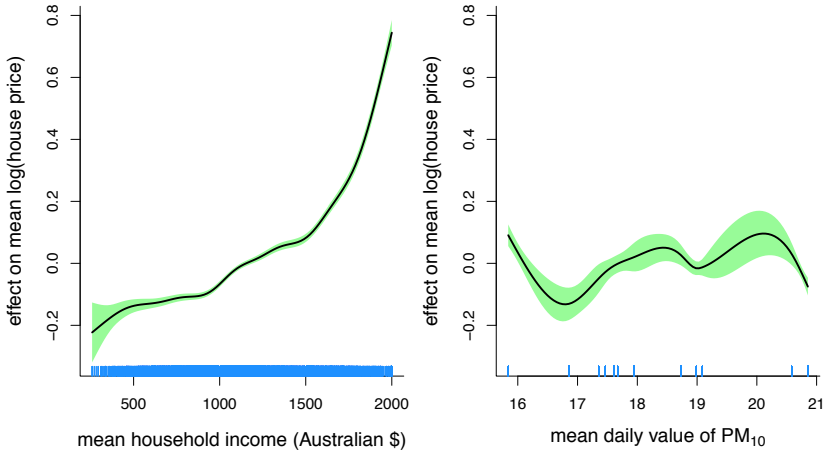
The HRW package in R has a demonstration named

SydneyDisplaySophis

that fits such a geoadditive model to data from

37,676 houses in OUR CITY.





Bivariate Penalized Splines in Assignment 7

See Assignment 7 for further details and Stan implementation of:

bivariate penalized splines

(including very cool 3D spin graphics).

♦ ♦ ♦

Penalized Splines for Bivariate Functions

Note to presenter: Run scallopFull.Rs (in sprSC)