

First Wanted: $p(x_2|x_1, x_3)$ (full conditional of x_2)

Answer:

$$\begin{aligned} p(x_3|x_1, x_2) &= \frac{p(x_1, x_2, x_3)}{p(x_1, x_2)} \\ &\propto p(x_1, x_2, x_3) \\ &= p(x_2|x_1, x_3)p(x_1)p(x_3) \\ &\propto p(x_2|x_1, x_3)p(x_3) \\ &\propto \frac{1}{\sqrt{\pi/x_3}} \exp\left\{-\frac{(x_2 - 47 + 11x_1)^2}{1/x_3}\right\} \frac{1}{\Gamma(6)} x_3^5 \exp(-x_3) \\ &\propto x_3^{11/2} \exp\left[-x_3\{(x_2 - 47 + 11x_1)^2 + 1\}\right] \\ &= x_3^{(13/2)-1} \exp\left[-x_3\{(x_2 - 47 + 11x_1)^2 + 1\}\right] \end{aligned}$$

By inspection of DEFINITION 2.4,

$$x_3|x_1, x_2 \sim \text{Gamma}\left(\frac{13}{2}, (x_2 - 47 + 11x_1)^2 + 1\right).$$