Second Wanted: $p(x_1|x_2, x_3)$ (full conditional of x_1)

Answer:

$$
\begin{aligned}
p(x_1|x_2, x_3) &= \frac{p(x_1, x_2, x_3)}{p(x_2, x_3)} \\
&\propto p(x_1, x_2, x_3) \\
&= p(x_2|x_1, x_3)p(x_1)p(x_3) \\
&\propto p(x_2|x_1, x_3)p(x_1) \\
&= \frac{1}{\sqrt{\pi/x_3}} \exp\left\{-\frac{(x_2 - 47 + 11\,x_1)^2}{1/x_3}\right\} \frac{1}{\sqrt{\frac{1}{2}\pi}} \exp\left\{-\frac{(x_1 - 3)^2}{\frac{1}{2}}\right\} \\
&\propto \exp\left\{-x_3(x_2 - 47 + 11\,x_1)^2 - 2(x_1 - 3)^2\right\}
\end{aligned}
$$

$$
\begin{aligned}
-2\log\, p(x_1|x_2, x_3) &= 2x_3\{11\,x_1 + (x_2 - 47)\}^2 + 4(x_1 - 3)^2 + \text{const} \\
&= 2x_3\{121\,x_1^2 + 22\,x_1(x_2 - 47)\} + 4(x_1^2 - 6x_1) + \text{const} \\
&= 2(121x_3 + 2)x_1^2 + 2\{22x_3(x_2 - 47) - 12\}x_1 + \text{const}
\end{aligned}
$$

With respect to Result 2.5 in the notes we have:

$$A = 2(121x_3 + 2) \quad \text{and} \quad B = 2\{22x_3(x_2 - 47) - 12\},$$

which gives

$$\frac{-B}{2A} = \frac{12 - 22x_3(x_2 - 47)}{121x_3 + 2}.$$

and so

$$x_1|x_2, x_3 \sim N\left(\frac{12 - 22x_3(x_2 - 47)}{121x_3 + 2}, \frac{1}{2(121\,x_3 + 2)}\right).$$