37458 Advanced Bayesian Methods

FINDING $p(y|x)$ FOR

ASSIGNMENT 1, QUESTION 4

THE TRADITIONAL WAY

$$p(x,y) = \frac{e^{-y(x^2+1)}}{\pi}, \quad x \in \mathbb{R},\ y > 0.$$

The marginal density function of x is

$$p(x) = \int_{-\infty}^{\infty} p(x,y)\,dy$$

$$= \frac{1}{\pi}\int_0^{\infty} e^{-y(x^2+1)}\,dy$$

$$= \frac{1}{\pi}\left[-\frac{e^{-y(x^2+1)}}{x^2+1}\right]_0^{\infty}$$

$$= \frac{1}{\pi(x^2+1)}, \quad x \in \mathbb{R}.$$

$$\Rightarrow \quad p(y|x) = \frac{p(x,y)}{p(x)} = \frac{\frac{1}{\pi}e^{-y(x^2+1)}}{\frac{1}{\pi}(x^2+1)^{-1}}$$

$$= (x^2+1)\,e^{-y(x^2+1)} \qquad x \in \mathbb{R},\ y > 0$$

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A SLICKER WAY ("BY INSPECTION")

$$p(y|x) = \frac{p(x,y)}{p(x)}$$

$$\propto \quad p(x,y)$$

$$= \frac{e^{-y(x^2+1)}}{\pi}$$

$$\propto \quad e^{-y(x^2+1)}, \quad y > 0$$

where "$\propto$" denotes proportionality between both sides as a function of y

From DEFINITION 2.4 of the "Graph Theory and Statistics" notes, as function of y $\quad e^{-y(x^2+1)}$

is a special case of the Gamma family with

$$A = 1 \quad \text{and} \quad B = x^2+1$$

So, by inspection,

$$p(y|x) = \frac{(x^2+1)^1}{\Gamma(1)} \, y^{1-1} \, e^{-(x^2+1)y}, \quad y > 0$$

$$= (x^2+1)\, e^{-(x^2+1)y}, \quad x \in \mathbb{R}, \; y > 0$$

$$\boxed{\text{IMPORTANT: No integration done here!}}$$